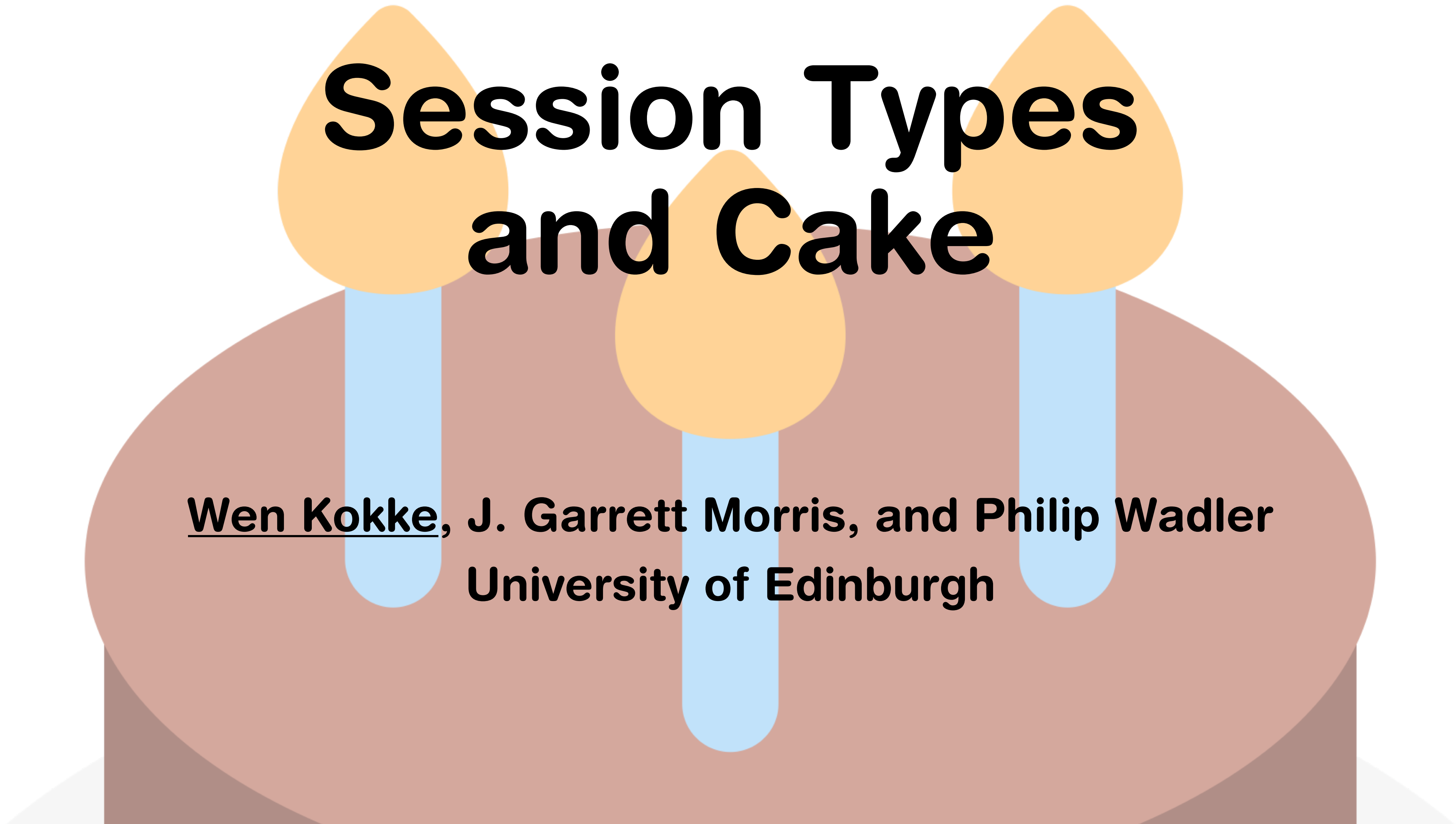
Session Types and Cake



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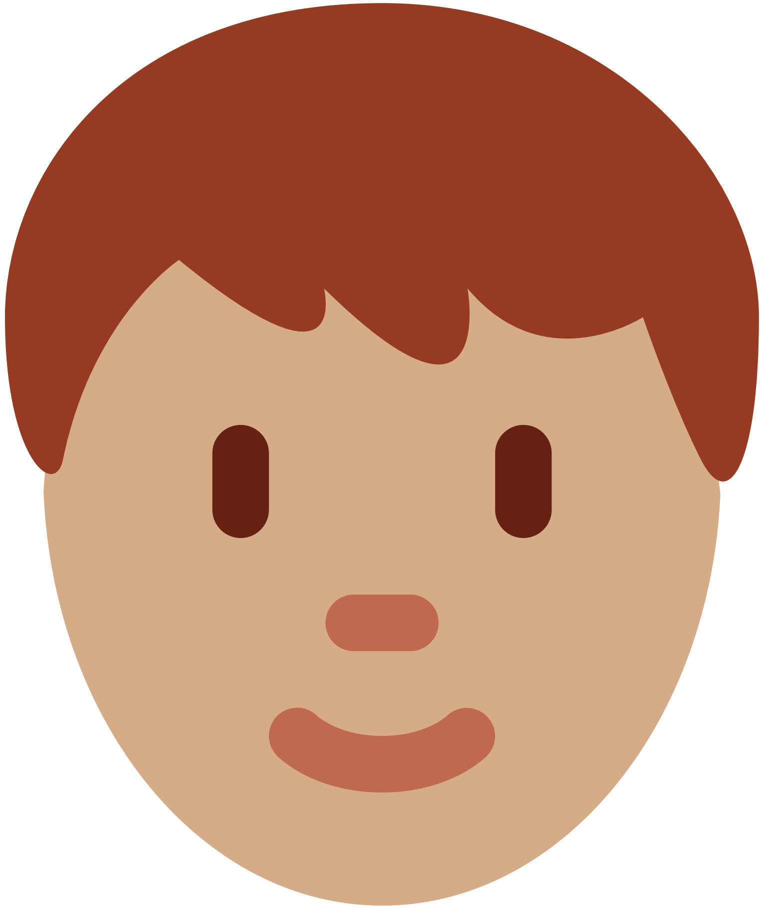
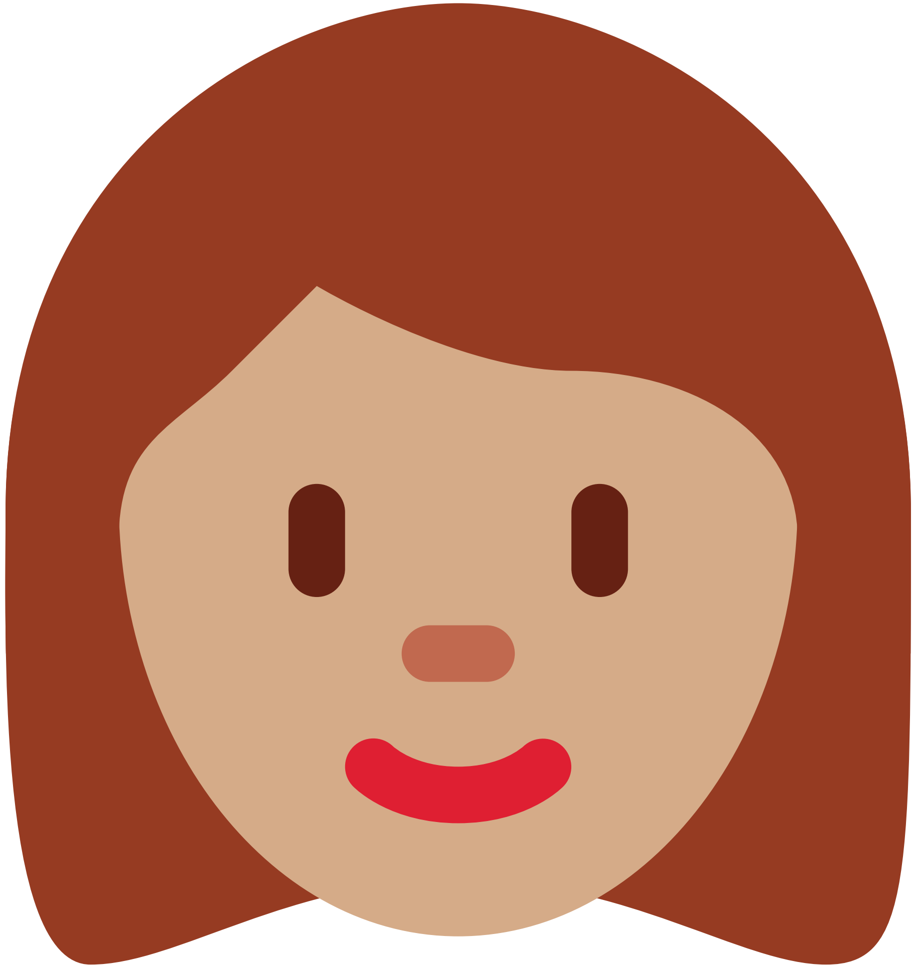
First, a story.



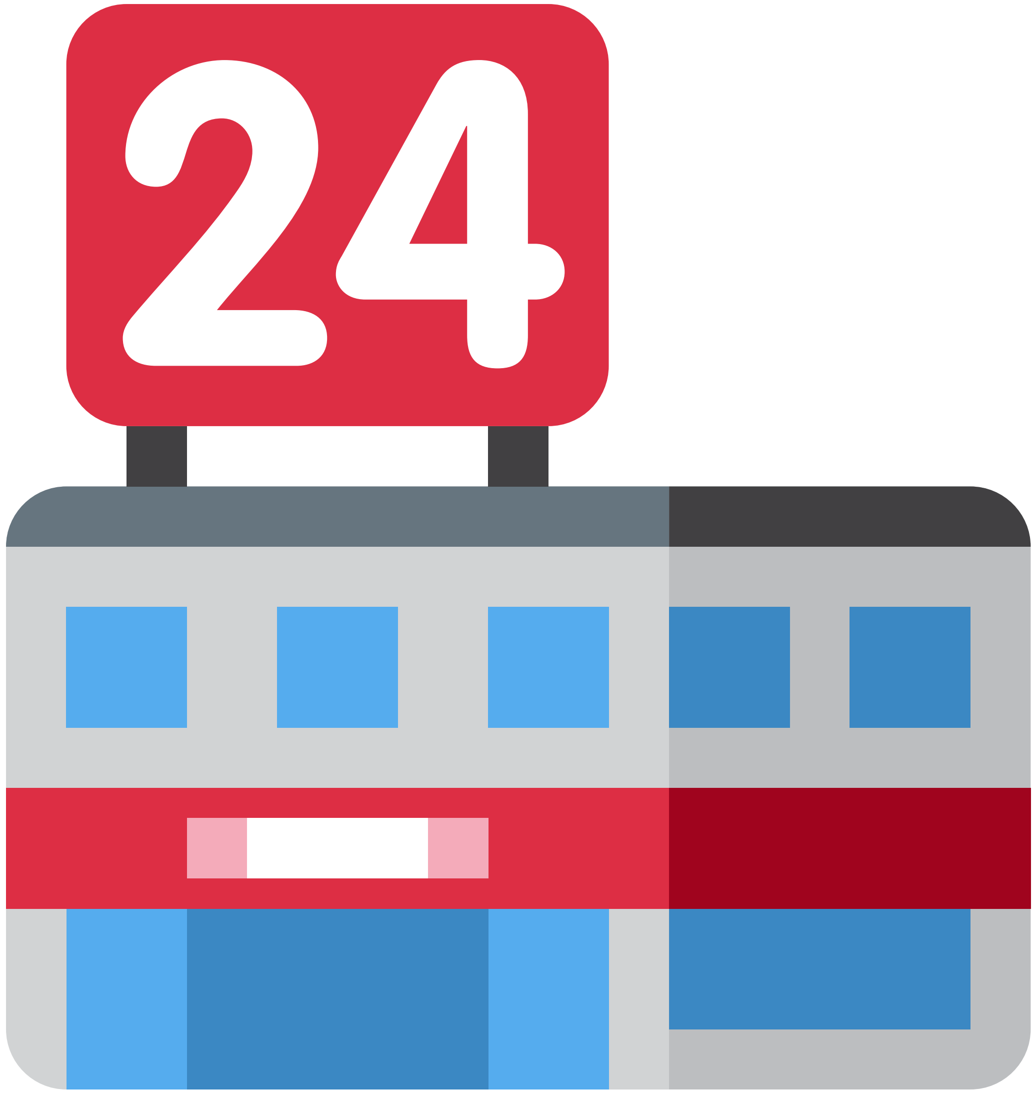
This is Ami.
They love cake.



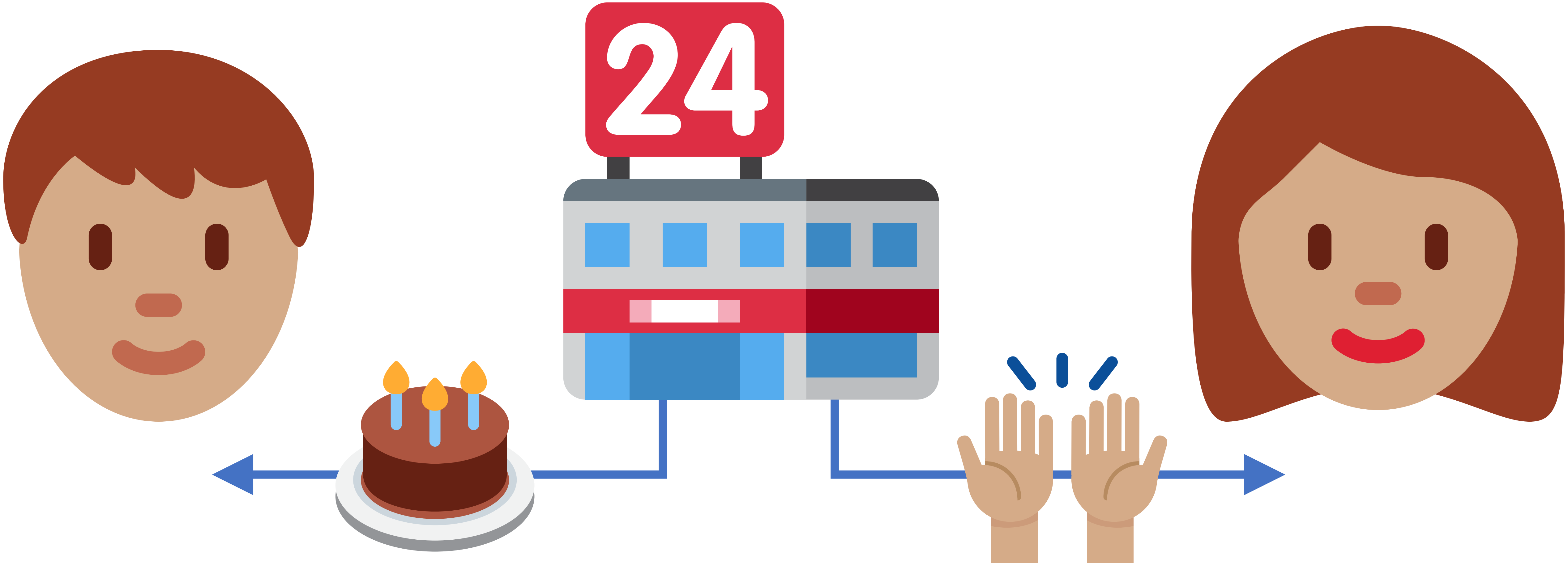
This is Boé.
She loves cake too.

This is a store.
It sells cake.

There is only
one cake left.

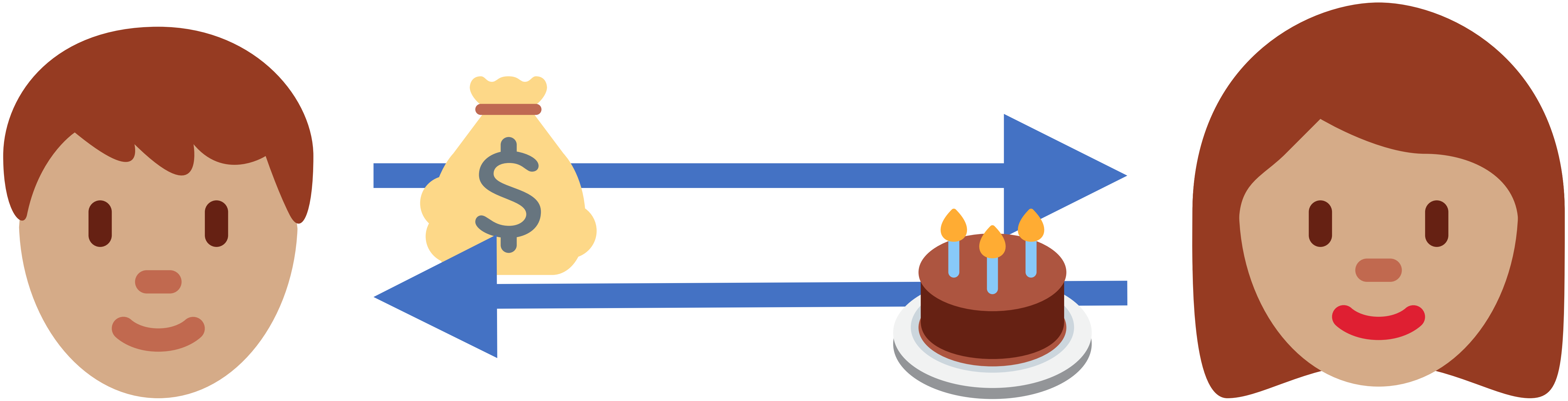


Ami and Boé have to *race*.
That's ok. The store doesn't mind.



So…
Races are good sometimes!

And deadlocks are bad.
I’m sure we all know.



This is Classical Processes.

$$\frac{}{x{\leftrightarrow}y \vdash x : A, y : A^{\perp}}\ \textsc{Ax} \qquad \frac{P \vdash \Gamma, x : A \qquad Q \vdash \Delta, y : A^{\perp}}{(\nu xy)(P \mid Q) \vdash \Gamma, \Delta}\ \textsc{Cut}$$

$$\frac{P \vdash \Gamma, y : A \qquad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}\ (\otimes) \qquad \frac{P \vdash \Gamma, y : A, x : B}{x(y).P \vdash \Gamma, x : A \parr B}\ (\parr)$$

$$\frac{}{x[].0 \vdash x : \mathbf{1}}\ (\mathbf{1}) \qquad \frac{P \vdash \Gamma}{x().P \vdash \Gamma, x : \perp}\ (\perp)$$

$$\frac{P \vdash \Gamma, x : A}{x \triangleleft \texttt{inl}.P \vdash \Gamma, x : A \oplus B}\ (\oplus_1) \qquad \frac{P \vdash \Gamma, x : B}{x \triangleleft \texttt{inr}.P \vdash \Gamma, x : A \oplus B}\ (\oplus_2)$$

$$\frac{P \vdash \Gamma, x : A \qquad Q \vdash \Gamma, x : B}{x \triangleright \{\texttt{inl} : P; \texttt{inr} : Q\} \vdash \Gamma, x : A \,\&\, B}\ (\&)$$

This is Classical Processes.

It’s basically classical linear logic,
typing a process calculus.

This is CP's syntax.

P, Q	$::=$	$(\nu xy)(P \mid Q)$	**We can make new channels *and split*.**
	$\mid$	$x[y].(P \mid Q)$	**We can send a message *and split*.**
	$\mid$	$x(y).P$	**We can receive a message.**
	$\mid$	$\ldots$	

Oh no! Something's wrong.

$(\nu xy)(\quad (x[y].$👦 $\mid x[z].$👩$) \quad \mid \quad y($🎂$).y($🙌$).$🏪 $\quad)$

Oh no! Something's wrong.

$$(x[y].\text{👦} \mid x[z].\text{👩})$$

pool of clients

No parallel composition?!

$$\frac{P \vdash \Gamma, x : A \qquad Q \vdash \Delta, y : A^{\perp}}{(\nu xy)(P \mid Q) \vdash \Gamma, \Delta}\ \textsc{Cut}$$

$$\frac{P \vdash \Gamma, y : A \qquad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}\ (\otimes)$$

Just doing it is dangerous!

$$\frac{P \vdash \Gamma \qquad Q \vdash \Delta}{P \mid Q \vdash \Gamma, \Delta}\ \textsc{Mix}$$

$$\frac{P \vdash \Gamma, x : A, y : A^{\perp}}{(\nu xy)P \vdash \Gamma}\ \textsc{Cut} \qquad \frac{P \vdash \Gamma, y : A, x : B}{x[y].P \vdash \Gamma, x : A \otimes B}\ (\otimes)$$

Remember what's in parallel!

$$\mathcal{G}, \mathcal{H} \quad ::= \quad \varnothing \quad | \quad \Gamma \mid \mathcal{G}$$

$$\frac{P \vdash \mathcal{G} \qquad Q \vdash \mathcal{H}}{P \mid Q \vdash \mathcal{G} \mid \mathcal{H}} \textsc{H-Mix}$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : A \mid \Delta, y : A^{\perp}}{(\nu xy)P \vdash \mathcal{G} \mid \Gamma, \Delta} \textsc{Cut} \qquad \frac{P \vdash \Gamma, y : A \mid \Delta, x : B}{x[y].P \vdash \Gamma, \Delta, x : A \otimes B} (\otimes)$$

This is Hypersequent CP.

$$\frac{}{x{\leftrightarrow}y \vdash x : A, y : A^\perp}\ \textsc{Ax} \qquad \frac{P \vdash \mathcal{G} \mid \Gamma, x : A \mid \Delta, y : A^\perp}{(\nu xy)P \vdash \mathcal{G} \mid \Gamma, \Delta}\ \textsc{Cut} \qquad \frac{P \vdash \mathcal{G} \qquad Q \vdash \mathcal{H}}{P \mid Q \vdash \mathcal{G} \mid \mathcal{H}}\ \textsc{H-Mix}$$

$$\frac{P \vdash \Gamma, y : A \mid \Delta, x : B}{x[y].P \vdash \Gamma, \Delta, x : A \otimes B}\ (\otimes) \qquad \frac{P \vdash \Gamma, y : A, x : B}{x(y).P \vdash \Gamma, x : A \parr B}\ (\parr)$$

$$\frac{}{x[].0 \vdash x : \mathbf{1}}\ (\mathbf{1}) \qquad \frac{P \vdash \Gamma}{x().P \vdash \Gamma, x : \bot}\ (\bot)$$

$$\frac{P \vdash \Gamma, x : A}{x \triangleleft \texttt{inl}.P \vdash \Gamma, x : A \oplus B}\ (\oplus_1) \qquad \frac{P \vdash \Gamma, x : B}{x \triangleleft \texttt{inr}.P \vdash \Gamma, x : A \oplus B}\ (\oplus_2)$$

$$\frac{P \vdash \Gamma, x : A \qquad Q \vdash \Gamma, x : B}{x \triangleright \{\texttt{inl} : P; \texttt{inr} : Q\} \vdash \Gamma, x : A \,\&\, B}\ (\&)$$

This is Hypersequent CP.

It’s still basically classical linear logic, typing a process calculus.

Except now it has parallel composition.

This is HCP’s syntax.

$$
\begin{array}{lcl}
P, Q & ::= & (\nu x y)P \\
 & \mid & P \mid Q \\
 & \mid & x[y].P \\
 & \mid & x(y).P \\
 & \mid & \ldots
\end{array}
$$

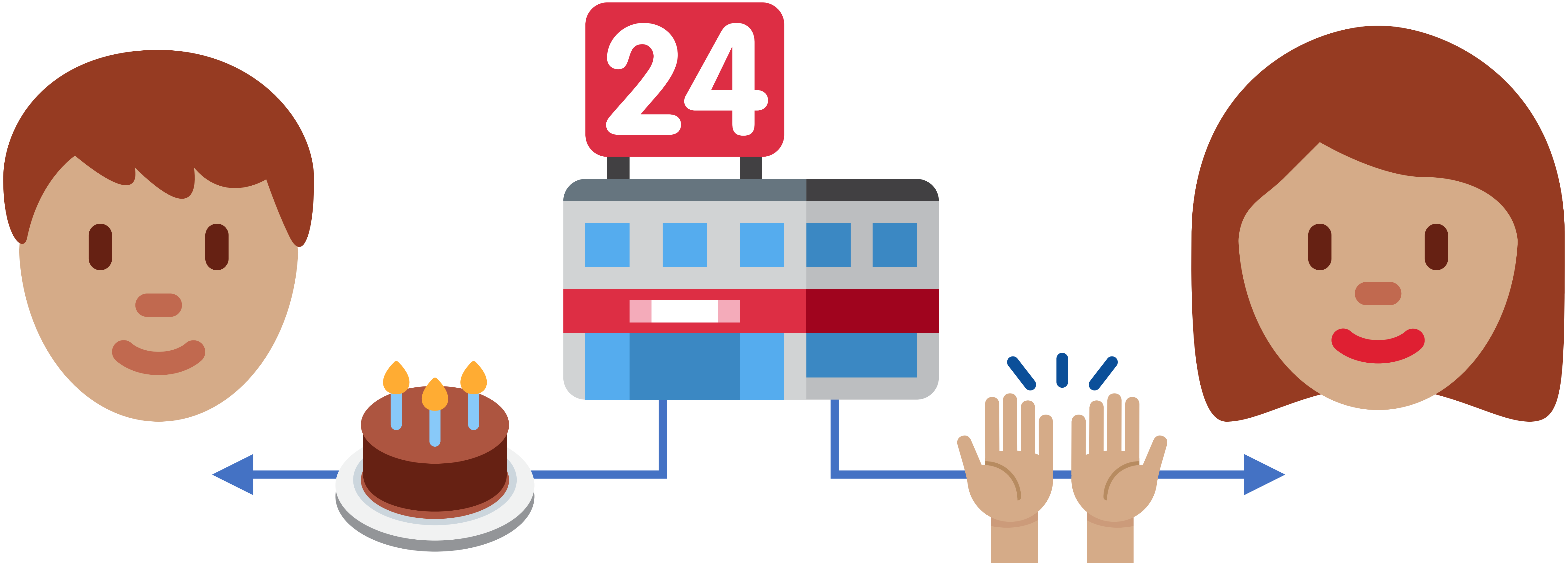
We can make new channels.

We can split.

We can send message y over channel x.

We can receive message y over channel x.

Ami and Boé have to *race*.
That's ok. The store doesn't mind.



This is what we've added.

$$\frac{P \vdash \Gamma, y : A}{\star x[y].P \vdash \Gamma, x : !_1 A} \; (!_1)$$

$$\frac{P \vdash \Gamma, y : A}{\star x(y).P \vdash \Gamma, x : ?_1 A} \; (?_1)$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : !_n A \mid \Delta, x' : !_m A}{P\{x/x'\} \vdash \mathcal{G} \mid \Gamma, \Delta, x : !_{n+m} A} \; \textsc{Cont}_!$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : ?_n A, x' : ?_m A}{P\{x/x'\} \vdash \mathcal{G} \mid \Gamma, x : ?_{n+m} A} \; \textsc{Cont}_?$$

This is our example race.

$$\dfrac{👦 \vdash \Gamma, z : 🎁}{\star x[z].👦 \vdash \Gamma, x : !_1 🎁}\ (!_1) \qquad \dfrac{👩 \vdash \Delta, w : 🎁}{\star x'[w].👩 \vdash \Delta, x' : !_1 🎁}\ (!_1)$$

$$\dfrac{\star x[z].👦 \vdash \Gamma, x : !_1 🎁 \qquad \star x'[w].👩 \vdash \Delta, x' : !_1 🎁}{\star x[z].👦 \mid \star x'[w].👩 \vdash \Gamma, x : !_1 🎁 \mid \Delta, x' : !_1 🎁}\ \text{H-MIX}$$

$$\dfrac{\star x[z].👦 \mid \star x'[w].👩 \vdash \Gamma, x : !_1 🎁 \mid \Delta, x' : !_1 🎁}{\star x[z].👦 \mid \star x[w].👩 \vdash \Gamma, \Delta, x : !_2 🎁}\ \text{CONT}_!$$

$$\dfrac{\dfrac{\dfrac{🏪 \vdash \Theta, 🎂 : 🎁, 🙌 : 🎁}{\star y'(🙌).🏪 \vdash \Theta, 🎂 : 🎁, y' : ?_1 🎁}\ (?_1)}{\star y(🎂).\star y'(🙌).🏪 \vdash \Theta, y : ?_1 🎁, y' : ?_1 🎁}\ (?_1)}{\star y(🎂).\star y(🙌).🏪 \vdash \Theta, y : ?_2 🎁}\ \text{CONT}_?$$

$$\dfrac{\star x[z].👦 \mid \star x[w].👩 \vdash \Gamma, \Delta, x : !_2 🎁 \qquad \star y(🎂).\star y(🙌).🏪 \vdash \Theta, y : ?_2 🎁}{\star x[z].👦 \mid \star x[w].👩 \mid \star y(🎂).\star y(🙌).🏪 \vdash \Gamma, \Delta, x : !_2 🎁 \mid \Theta, y : ?_2 🎁}\ \text{H-MIX}$$

$$\dfrac{\star x[z].👦 \mid \star x[w].👩 \mid \star y(🎂).\star y(🙌).🏪 \vdash \Gamma, \Delta, x : !_2 🎁 \mid \Theta, y : ?_2 🎁}{(\nu xy)(\star x[z].👦 \mid \star x[w].👩 \mid \star y(🎂).\star y(🙌).🏪) \vdash \Gamma, \Delta, \Theta}\ \text{CUT}$$

(🎁: Could be cake, could be disappointing.)

This is our example race.

👦 $\vdash \Gamma, z :$ 🎁

👩 $\vdash \Delta, w :$ 🎁

🏪 $\vdash \Theta,$ 🎂 $:$ 🎁, 🙌 $:$ 🎁

(🎁: Could be cake, could be disappointing.)

Ami gets the cake.

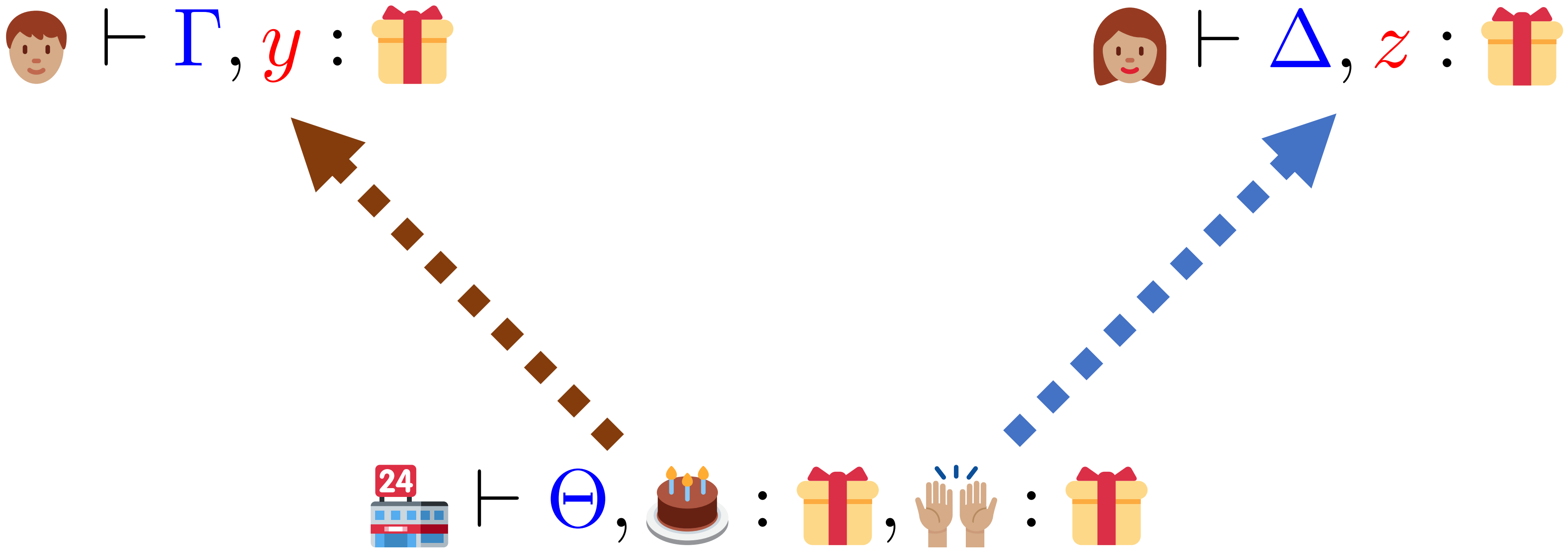
👦 $\vdash \Gamma, y :$ 🎁

👩 $\vdash \Delta, z :$ 🎁

🏪 $\vdash \Theta,$ 🎂 $:$ 🎁, 🙌 $:$ 🎁

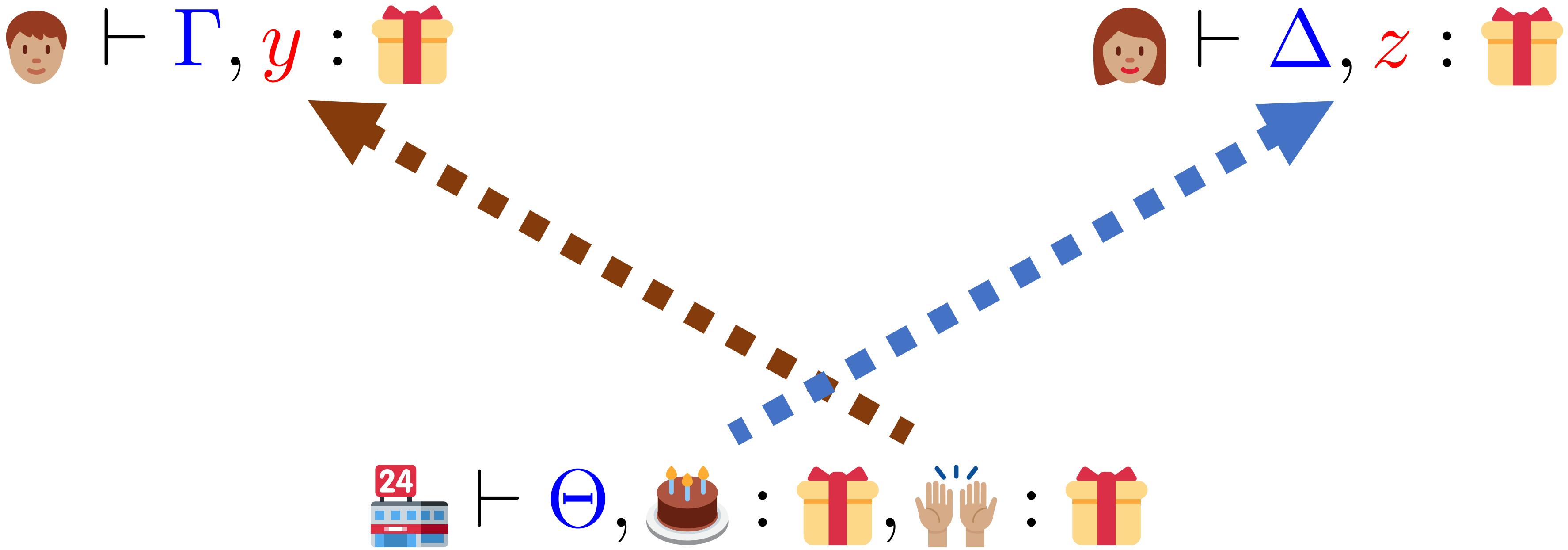
(🎁: Could be cake, could be disappointing.)

Ami gets the cake.



(🎁: Could be cake, could be disappointing.)

Boé gets the cake.



(🎁: Could be cake, could be disappointing.)

And so we had races, but no deadlocks.

What’s not yet right?

- No recursion 😔
- No infinite interactions 😩

But:
Conflation confers concurrency.

What is 'conflation'?

- Makes two duals isomorphic
- Conflation of !/? confers:
  - No lock freedom 😩
  - No termination 😔
  - Concurrent shared state 🙂
  - Recursion 😄

Other mechanisms?

- Non-deterministic local choice

$$P + Q \longrightarrow P$$
$$P + Q \longrightarrow Q$$

- Equally expressive, not equal
- Translate from O(1), to O(n!)

Other mechanisms?

Manifest Sharing:

- Recursion 🙂
- No deadlock freedom 😔
- Cannot interleave requests 😩

Non-deterministic HCP:

- Simple extension of HCP
- Finite non-determinism
- Deadlock free

Thanks!