

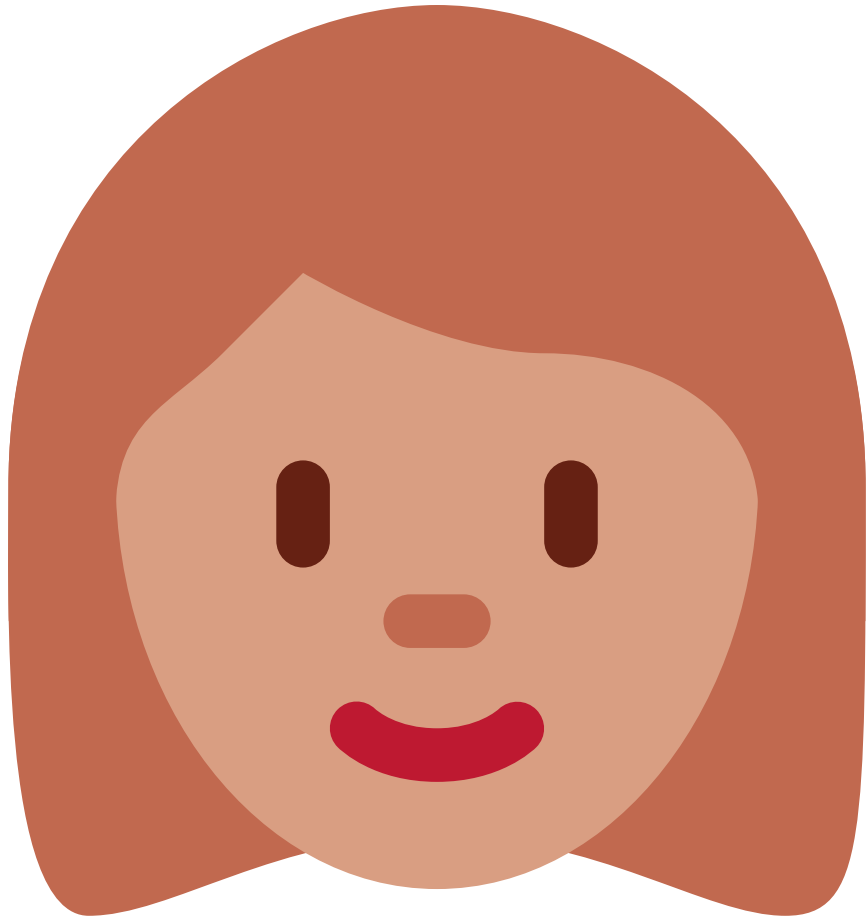
A stylized illustration of a cake with a light brown base and a darker brown top layer. Three light blue vertical stripes represent candles, each topped with a light orange flame. The title 'Session Types and Cake' is centered over the cake.

# **Session Types and Cake**

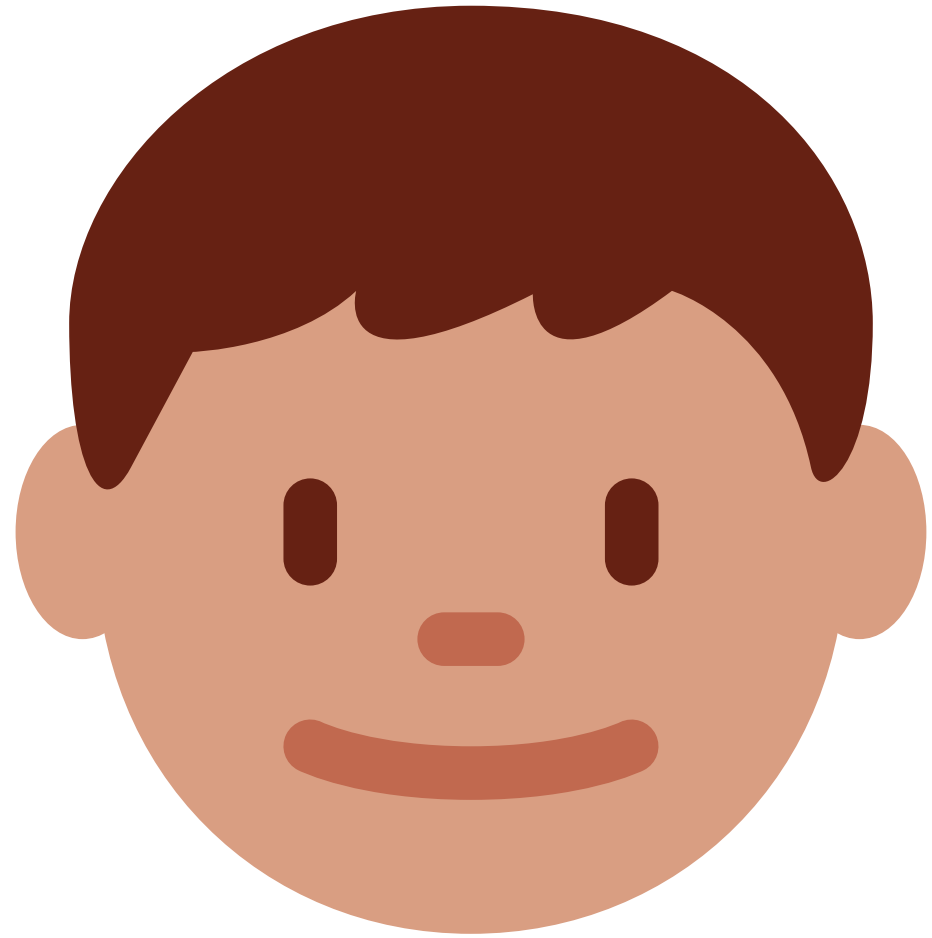
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**University of Edinburgh**

**Supervised by Phil Wadler and J. Garrett Morris**

**First, a story.**



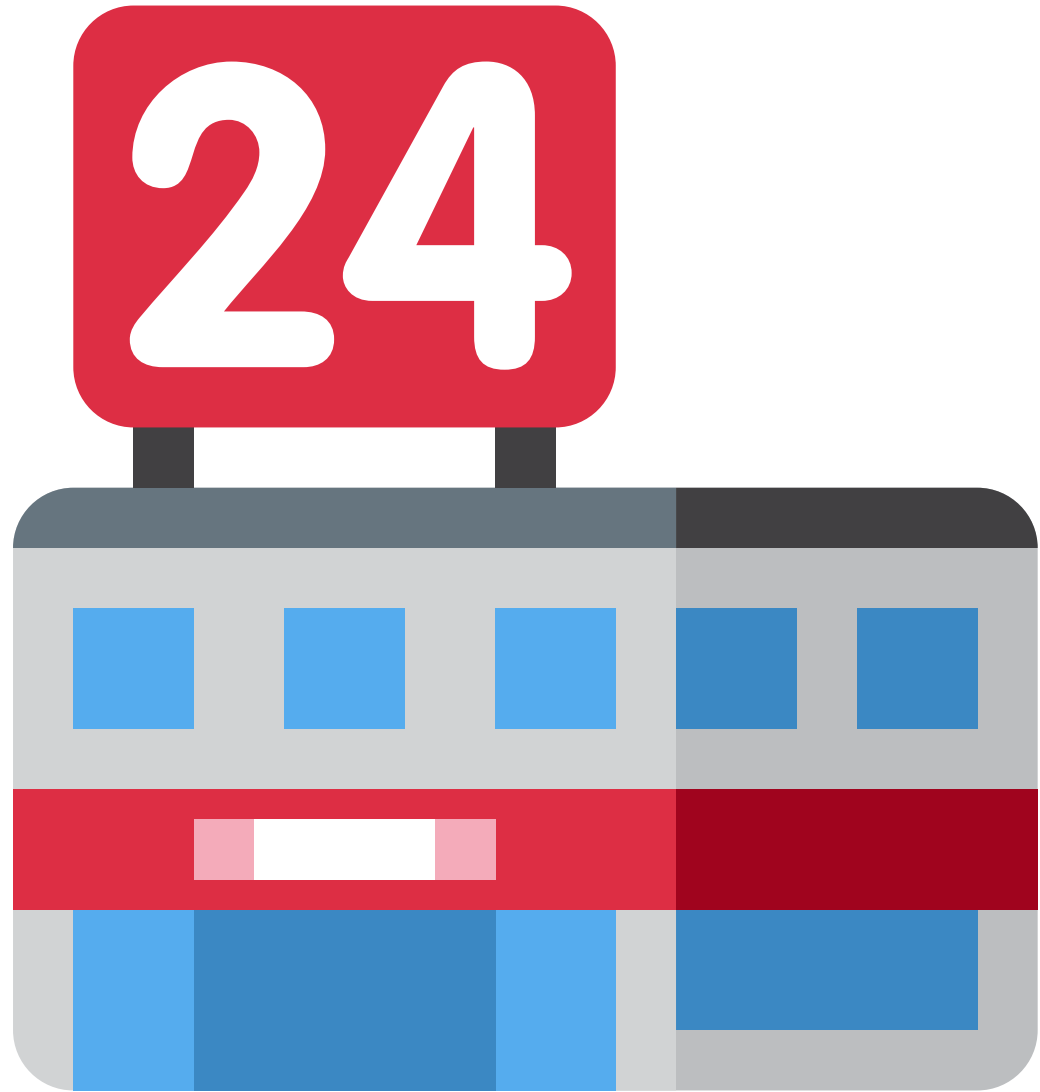
**This is Mary.  
She loves cake.**



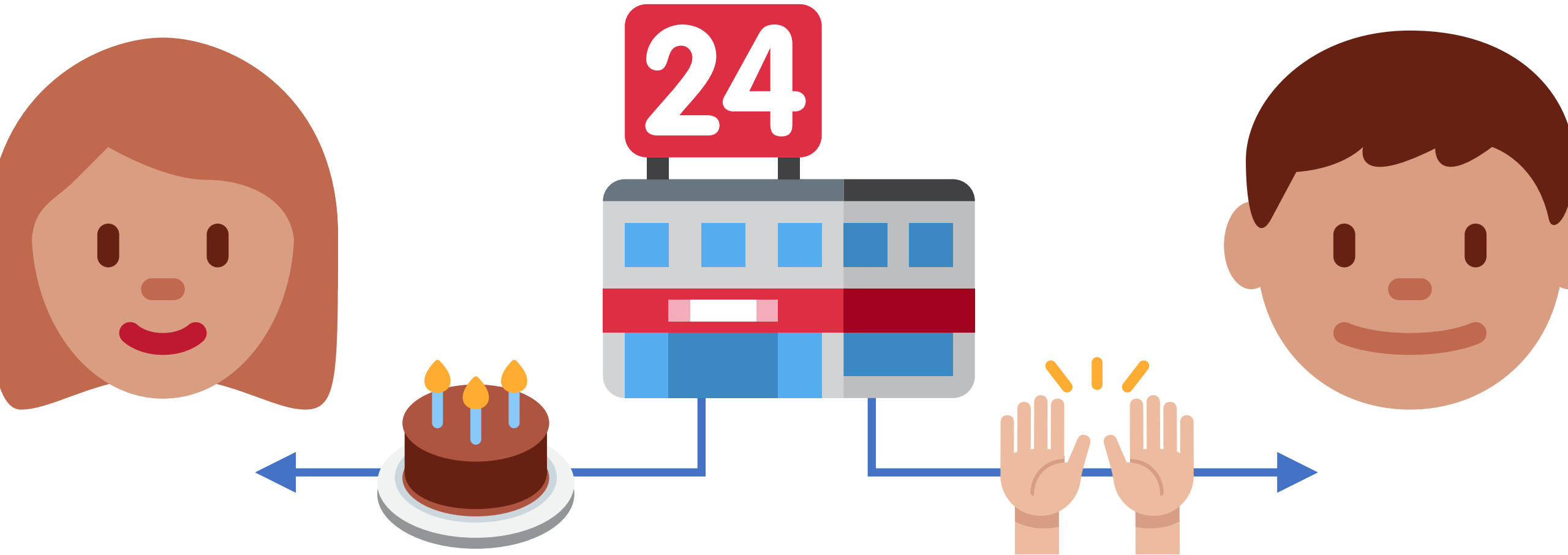
**This is John.  
He loves cake too.**

**This is a store.  
It sells cake.**

**There is only  
*one* cake left.**



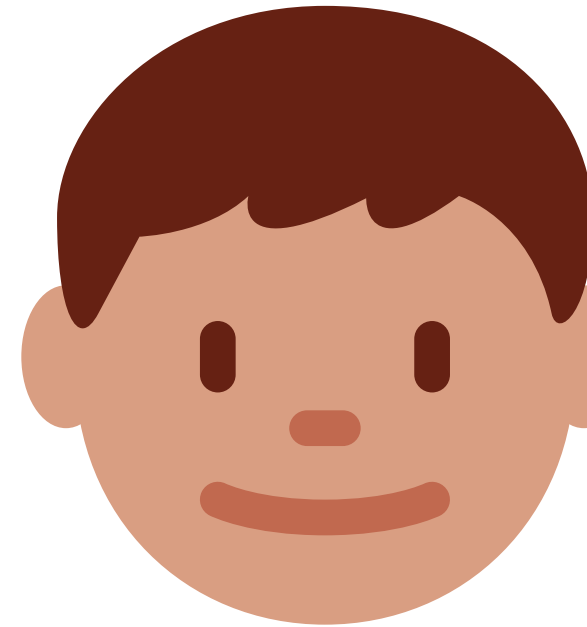
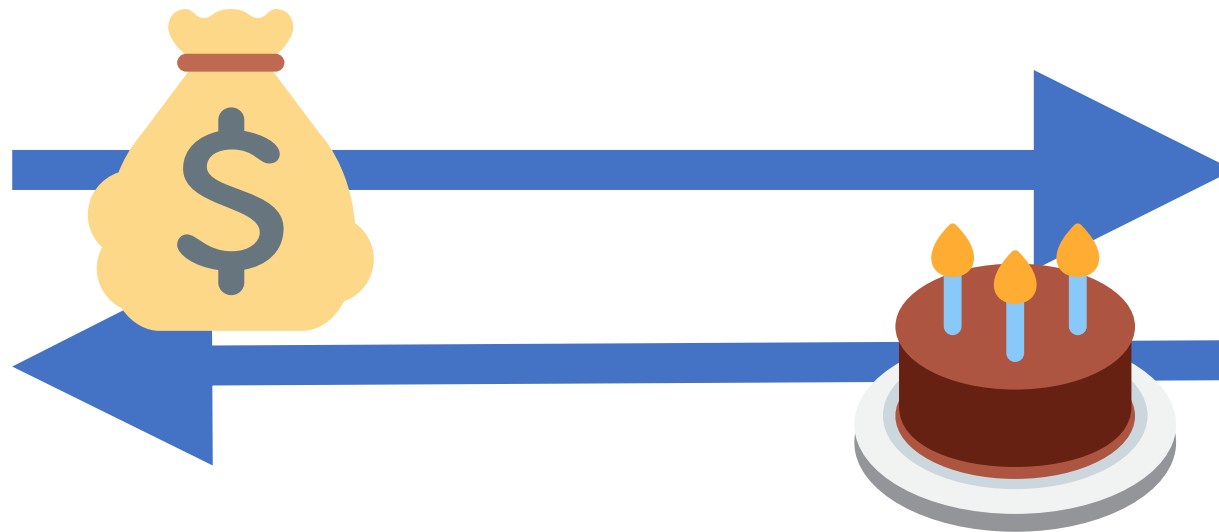
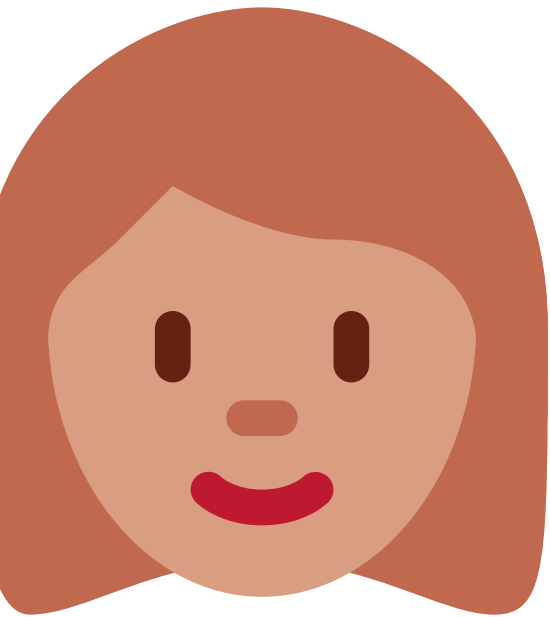
Mary and John have to *race*.  
That's ok. The store doesn't mind.



**So...**

**Races are good  
sometimes!**

**And deadlocks are bad.  
I'm sure we all know.**



# This is Classical Processes.

$$\frac{}{x \leftrightarrow y \vdash x:A, y:A^\perp} \text{Ax} \quad \frac{P \vdash \Gamma, x:A \quad Q \vdash \Delta, x:A^\perp}{\nu x.(P \mid Q) \vdash \Gamma, \Delta} \text{Cut}$$

$$\frac{P \vdash \Gamma, y:A \quad Q \vdash \Delta, x:B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x:A \otimes B} (\otimes) \quad \frac{P \vdash \Gamma, y:A, x:B}{x(y).P \vdash \Gamma, x:A \wp B} (\wp)$$

$$\frac{}{x[].0 \vdash x:\mathbf{1}} (\mathbf{1}) \quad \frac{P \vdash \Gamma}{x().P \vdash \Gamma, x:\perp} (\perp)$$

$$\frac{P \vdash \Gamma, x:A}{x[\text{inl}].P \vdash \Gamma, x:A \oplus B} (\oplus_1) \quad \frac{P \vdash \Gamma, x:B}{x[\text{inr}].P \vdash \Gamma, x:A \oplus B} (\oplus_2)$$

$$\frac{P \vdash \Gamma, x:A \quad Q \vdash \Delta, x:B}{\text{case } x \{P; Q\} \vdash \Gamma, \Delta, x:A \& B} (\&) \quad \frac{}{\text{case } x \{ \} \vdash \Gamma, x:\top} (\top)$$



# This is Classical Processes.

It's basically classical linear logic,  
typing a process calculus.

$$\begin{array}{c}
 \frac{}{x \multimap y \vdash x:A, y:A^\perp} \text{Ax} \quad \frac{P \vdash \Gamma, x:A \quad Q \vdash \Delta, x:A^\perp}{x \multimap (P \mid Q) \vdash \Gamma, \Delta} \text{Cut} \\
 \\
 \frac{P \vdash \Gamma, y:A \quad Q \vdash \Delta, x:B}{x[y](P \mid Q) \vdash \Gamma, \Delta, x:A \otimes B} (\otimes) \quad \frac{P \vdash \Gamma, y:A, x:B}{x[y].P \vdash \Gamma, x:A \wp B} (\wp) \\
 \\
 \frac{}{x \multimap () \vdash x:A} (\bot) \quad \frac{}{x[()]P \vdash \Gamma, x:A} (\top) \\
 \\
 \frac{P \vdash \Gamma, x:A}{x[\text{in}].P \vdash \Gamma, x:A \oplus B} (\oplus_1) \quad \frac{P \vdash \Gamma, x:B}{x[\text{in}].P \vdash \Gamma, x:A \oplus B} (\oplus_2) \\
 \\
 \frac{P \vdash \Gamma, x:A \quad Q \vdash \Delta, x:B}{\text{case } x (P; Q) \vdash \Gamma, \Delta, x:A \& B} (\&) \quad \frac{}{\text{case } x () \vdash \Gamma, x:\top} (\top)
 \end{array}$$

# This is what I've added.

$A, B, C := \dots$

|  $!_n A$  pool of  $n$  clients  
 |  $?_n A$   $n$  server interactions

$P, Q, R := \dots$

|  $\star x[y].P$  client creation  
 |  $\star x(y).P$  server interaction  
 |  $(P \mid Q)$  parallel composition of clients

$$\frac{P \vdash \Gamma, y:A}{\star x[y].P \vdash \Gamma, x: !_1 A} (!_1) \quad \frac{P \vdash \Gamma, y:A}{\star x(y).P \vdash \Gamma, x: ?_1 A} (?_1)$$

$$\frac{P \vdash \Gamma, x: !_m A \quad Q \vdash \Delta, x: !_n A}{(P \mid Q) \vdash \Gamma, \Delta, x: !_m A} \text{POOL} \quad \frac{P \vdash \Gamma, x: ?_m A, y: ?_n A}{P\{x/y\} \vdash \Gamma, x: ?_{m+n} A} \text{CONT}$$

# This is our example race.

$$\begin{array}{c}
 \frac{\frac{\text{👩} \vdash \Gamma, y: \text{🎁}^\perp}{\star x[y]. \text{👩} \vdash \Gamma, z: !_1 \text{🎁}^\perp} (!_1) \quad \frac{\frac{\text{👨} \vdash \Delta, z: \text{🎁}^\perp}{\star x[z]. \text{👨} \vdash \Delta, z: !_1 \text{🎁}^\perp} (!_1)}{(\star x[y]. \text{👩} \mid \star x[z]. \text{👨}) \vdash \Gamma, \Delta, x: !_2 \text{🎁}^\perp} \text{POOL} \\
 \frac{\quad \quad \quad \frac{\frac{\frac{\text{🏠}^{24} \vdash \Theta, \text{🍰}: \text{🎁}, \text{👏}: \text{🎁}}{\star x'(\text{👏}). \text{🏠}^{24} \vdash \Theta, \text{🍰}: \text{🎁}, x': ?_1 \text{🎁}} (?_1)}{\star x(\text{🍰}). \star x'(\text{👏}). \text{🏠}^{24} \vdash \Theta, x: ?_1 \text{🎁}, x': ?_1 \text{🎁}} (?_1)}{\star x(\text{🍰}). \star x(\text{👏}). \text{🏠}^{24} \vdash \Theta, x: ?_2 \text{🎁}} \text{CONT} \\
 \hline
 \nu x. ((\star x[y]. \text{👩} \mid \star x[z]. \text{👨}) \mid \star x(\text{🍰}). \star x(\text{👏}). \text{🏠}^{24}) \vdash \Gamma, \Delta, \Theta \quad \text{CUT}
 \end{array}$$

(🎁: Could be cake, could be disappointing.)

# This is our example race.


👩  $\vdash \Gamma, y: \text{🎁}^\perp$


👨  $\vdash \Delta, z: \text{🎁}^\perp$

📅  $\vdash \Theta, \text{🍰}: \text{🎁}, \text{👏}: \text{🎁}$

(🎁: Could be cake, could be disappointing.)

# Who gets the cake?

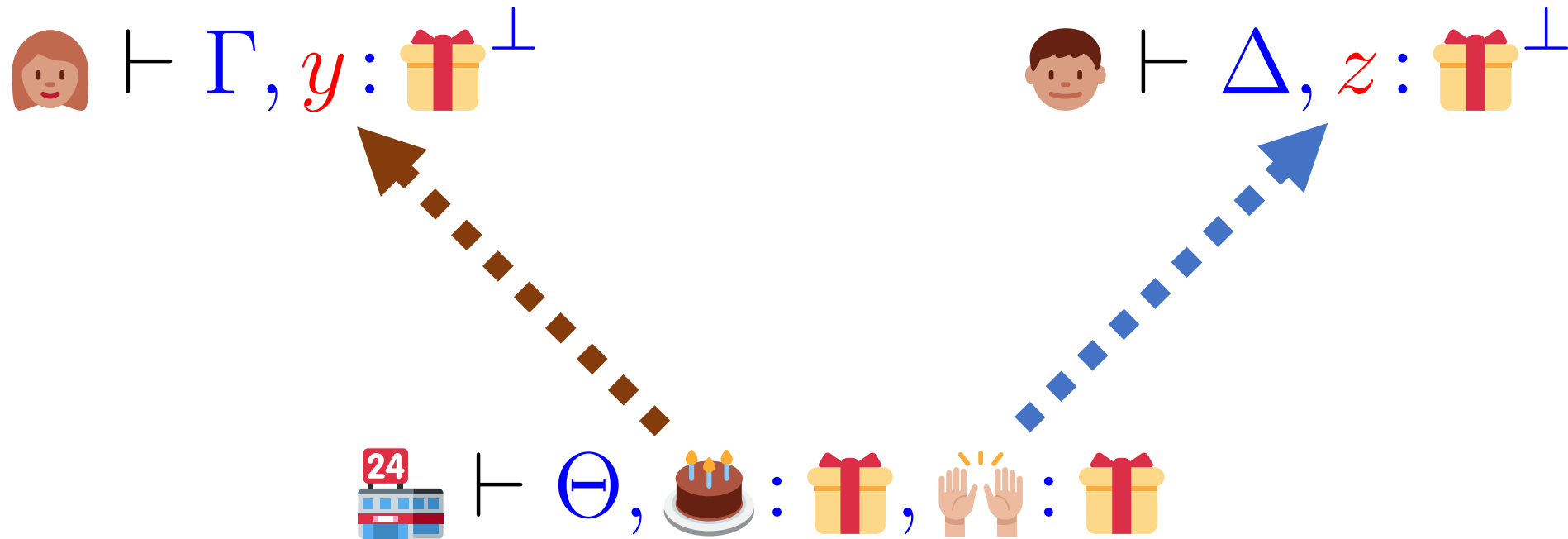
  $\vdash \Gamma, y : \text{🎁}^\perp$

  $\vdash \Delta, z : \text{🎁}^\perp$

  $\vdash \Theta, \text{🍰} : \text{🎁}, \text{👏} : \text{🎁}$

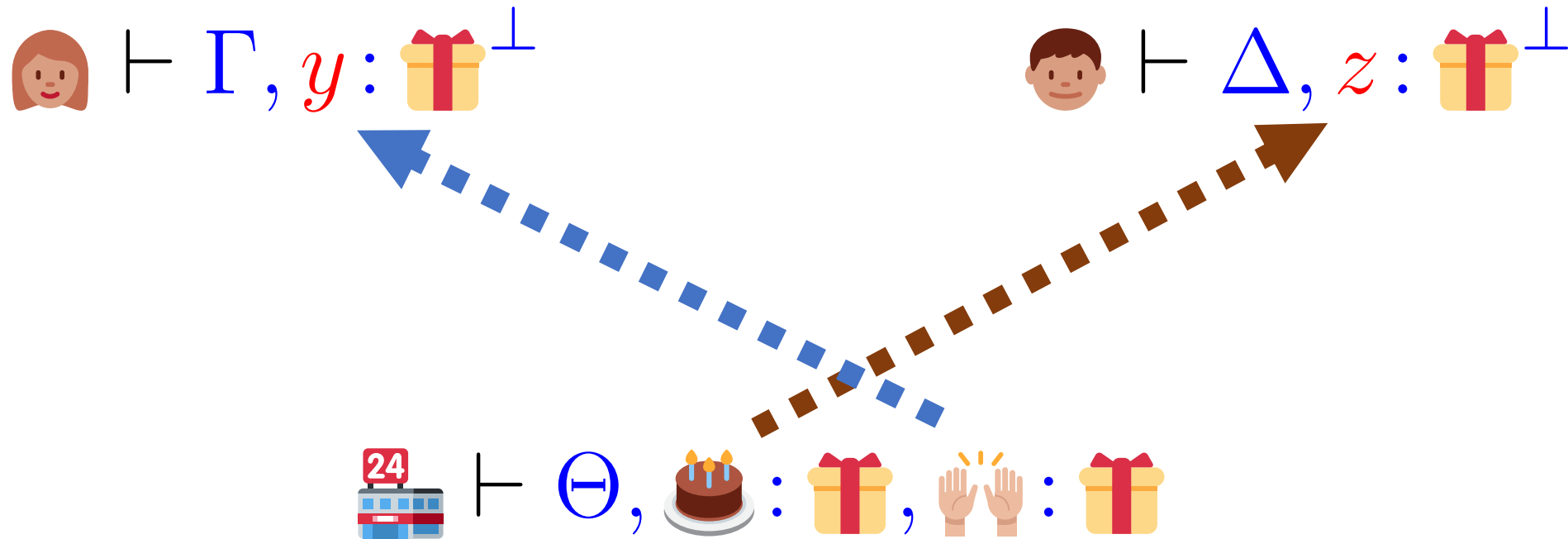
( : Could be cake, could be disappointing.)

# Mary gets the cake.



(📁: Could be cake, could be disappointing.)

# John gets the cake.



(🎁: Could be cake, could be disappointing.)

**And so we had races,  
but no deadlocks.**

**The End.**



# Thanks!



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