Taking Apart Classical Processes

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December 18th, 2017

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Dramatis personæ

👩 Mary

👦 John

🎂 Cake

💰 Money

Dramatis personæ

- 👩 Mary
- 👦 John
- 🎂 Cake
- 💰 Money

Dramatis personæ

- 👩 Mary
- 👦 John
- 🎂 Cake
- 💰 Money

Cake so Good It Should Be Illegal

"Gimme the cake, and you'll have your money!" "No! Money first!"

$$(\nu x)(\; x(z).x\langle 💰 \rangle.👩🏻 \;\mid\; x(y).x\langle 🎂 \rangle.👦🏾 \;)$$

(Milner, Parrow, and Walker, 1992.)

Classical Processes – Types, Contexts, and Typing Rules

$$\text{Type } A, B \;\; ::= A \otimes B \mid A \parr B \mid \ldots$$

$$\text{Ctxt } \;\; \Gamma, \Delta \;\; ::= x_1 : A_1, \ldots, x_n : A_n$$

$$\frac{}{x \leftrightarrow y \;\vdash\; x : A, y : A^\perp}\,\textsc{Ax} \qquad \frac{P \;\vdash\; \Gamma, x : A \qquad Q \;\vdash\; \Delta, x : A^\perp}{(\nu x)(P \mid Q) \;\vdash\; \Gamma, \Delta}\,\textsc{Cut}$$

(Caires and Pfenning, 2010|Wadler, 2012.)

$$\text{Type } A, B \;:= A \otimes B \mid A \parr B \mid \ldots$$
$$\text{Ctxt } \Gamma, \Delta \;:= x_1 : A_1, \ldots, x_n : A_n$$

$$\frac{P \vdash \Gamma, y : A \qquad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}(\otimes) \qquad \frac{P \vdash \Gamma, y : A, x : B}{x(y).P \vdash \Gamma, x : A \parr B}(\parr)$$

$$\frac{}{x[].0 \vdash x : 1}(1) \qquad \frac{P \vdash \Gamma}{x().P \vdash \Gamma, x : \bot}(\bot)$$

(Caires and Pfenning, 2010|Wadler, 2012.)

Classical Processes – Best Facilitator of Illegal Cake Resale

“Fine! I’ll go first!”

$$(\nu x)(\; x[u].(u \leftrightarrow \text{💰} \mid x(z).\text{👩🏻}) \mid \; x(y).x[v].(v \leftrightarrow \text{🎂} \mid \text{👦🏾}))$$

Classical Processes – Terms

$$
\begin{aligned}
P, Q, R := {} & (\nu x)(P \mid Q) && \textit{Communication} \\
\mid {} & x[y].(P \mid Q) && \textit{Independence, “send”} \\
\mid {} & x(y).P && \textit{Interdependence, “receive”} \\
\mid {} & x[].0 && \textit{Halt} \\
\mid {} & x().P && \textit{Wait} \\
\mid {} & \ldots
\end{aligned}
$$

Type

Ctxt

I'm gonna talk about hypersequent CP, about constrained cyclic Me

$$\frac{}{x{\leftrightarrow}y \ \vdash \ x : A, y : A^{\perp}}\ \textsc{Ax} \qquad \frac{}{0 \ \vdash \ \varnothing}\ \textsc{H-Halt}$$

$$\frac{P \ \vdash \ \mathcal{G} \mid \Gamma, x : A \mid \Delta, x : A^{\perp}}{(\nu x)P \ \vdash \ \mathcal{G} \mid \Gamma, \Delta}\ \textsc{H-Cut} \qquad \frac{P \ \vdash \ \mathcal{G} \qquad Q \ \vdash \ \mathcal{H}}{P \mid Q \ \vdash \ \mathcal{G} \mid \mathcal{H}}\ \textsc{H-Mix}$$

HCCP – An Intuition for Semicolons

If P is typed by $P \vdash \Gamma$,
it will reduce to a single process.

If P is typed by $P \vdash \Gamma_1 \mid \ldots \mid \Gamma_n$,
it will reduce to a series of n parallel processes.

HCCP – Cut is Derivable

$$\dfrac{\dfrac{P \vdash x : A, \Gamma \qquad Q \vdash \bar{x} : A^{\perp}, \Delta}{(P \mid Q) \vdash x : A, \Gamma \mid \bar{x} : A^{\perp}, \Delta}\ \textsc{Mix}}{(\nu x \bar{x})(P \mid Q) \vdash \Gamma, \Delta}\ \textsc{Cycle}$$

HCCP – Cut is Derivable

$$\dfrac{\dfrac{P \vdash x : A, \Gamma \mid X \qquad Q \vdash \bar{x} : A^{\perp}, \Delta \mid Y}{(P \mid Q) \vdash x : A, \Gamma \mid \bar{x} : A^{\perp}, \Delta \mid X \mid Y}\ \textsc{Mix}}{(\nu x\bar{x})(P \mid Q) \vdash \Gamma, \Delta \mid X \mid Y}\ \textsc{Cycle}$$

"Multicut"

$$\frac{P \vdash x_1 : A_1, \Gamma_1 \mid \ldots \mid x_n : A_n, \Gamma_n \mid X \qquad Q \vdash \bar{x}_1 : A_1^{\perp}, \Delta_1 \mid \ldots \mid \bar{x}_n : A_n^{\perp}, \Delta_n}{(\nu x_1 \bar{x}_1 \ldots x_n \bar{x}_n)(P \mid Q) \vdash \Gamma_1, \Delta_1 \mid \ldots \mid \Gamma_n, \Delta_n \mid X \mid Y}$$

$$\begin{array}{llll} \text{Type} & A, B & := & A \otimes B \mid A ⅋ B \mid \mathbf{1} \mid \bot \mid \ldots \\ \text{Ctxt} & \Gamma, \Delta & := & x_1 : A_1, \ldots, x_n : A_n \\ \text{Meta} & X, Y & := & \Gamma_1 \mid \ldots \mid \Gamma_n \end{array}$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, y : A \mid \Delta, x : B}{x[y].P \vdash \mathcal{G} \mid \Gamma, \Delta, x : A \otimes B} \otimes \qquad \frac{P \vdash \mathcal{G} \mid \Gamma, y : A, x : B}{x(y).P \vdash \mathcal{G} \mid \Gamma, x : A ⅋ B} (⅋)$$

$$\frac{P \vdash \mathcal{G}}{x[].P \vdash \mathcal{G} \mid x : \mathbf{1}} \mathbf{1} \qquad \frac{P \vdash \mathcal{G} \mid \Gamma}{x().P \vdash \mathcal{G} \mid \Gamma, x : \bot} (\bot)$$

HCCP – 41st Best Facilitator of Illegal Cake Resale

“Fine! I’ll go first!”

$$
\begin{aligned}
(\nu x\bar{x})(\; & x[u].(u\leftrightarrow \text{💰} \mid x(z).\text{👩🏻}) \\
& \mid \bar{x}(y).\bar{x}[v].(v\leftrightarrow \text{🎂} \mid \text{👦🏾})\;)
\end{aligned}
$$

HCCP – Terms

$$
\begin{aligned}
P, Q, R := {} & (\nu x\bar{x})P && \textit{ChannelCreation} \\
\mid {} & (P \mid Q) && \textit{ParallelComposition} \\
\mid {} & 0 && \textit{HaltedProcess} \\
\mid {} & x[y].P && \textit{Independence, “send”} \\
\mid {} & x(y).P && \textit{Interdependence, “receive”} \\
\mid {} & \ldots
\end{aligned}
$$

HCCP – Setting Send Free

$$x[y].P := (\nu y\bar{y})(x\langle y\rangle.P)$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : B}{x\langle y\rangle.P \vdash \mathcal{G} \mid \Gamma, x : A \otimes B, y : A^{\perp}} \otimes$$

$$\dfrac{\dfrac{P \vdash y : A, \Gamma \mid x : B, \Delta \mid X}{x\langle y\rangle.P \vdash y : A, \Gamma \mid x : A \otimes B, \bar{y} : A^{\perp}, \Delta \mid X} \otimes}{(\nu y\bar{y})(x\langle y\rangle.P) \vdash x : A \otimes B, \Gamma, \Delta \mid X} \textsc{Cycle}$$

(Boreale, 1996|Lindley and Morris, 2015.)

Cake so Good It Should Be Illegal

"Fine! I'll go first!"

$$(\nu x)(\; x\langle \text{💰} \rangle.x(z).\text{👩🏻} \;\mid\; x(y).x\langle \text{🎂} \rangle.\text{👦🏾} \;)$$

We have taken CP apart, and its term constructs now match that of the π-calculus, more or less*!

(*Modulo *a lot of things*, like the case construct, etc.)

If $P \vdash \Gamma_1 \mid \ldots \mid \Gamma_{n+1}$,
then there exist $x_1 \ldots x_n$ and $\pi_1 \ldots \pi_{n+1}$, such that

$$(\nu x_1 \bar{x}_1) \ldots (\nu x_n \bar{x}_n)(\pi_1.0 \mid \cdots \mid \pi_{n+1}.0) \vdash \Gamma_1 \mid \ldots \mid \Gamma_{n+1}.$$

If $P \vdash \Gamma_1 \mid \ldots \mid \Gamma_{n+1}$,
then there exist $x_1 \ldots x_n$ and $\pi_1 \ldots \pi_{n+1}$, such that

$$(\nu x_1 \bar{x}_1) \ldots (\nu x_n \bar{x}_n)(\{\pi_1, \ldots, \pi_{n+1}\}.(0^{n+1})) \vdash \Gamma_1 \mid \ldots \mid \Gamma_{n+1},$$

where (0^n) represents n halted processes in parallel.

So, for instance

$$(\nu x \bar{x})(x().0 \mid \bar{x}\langle\rangle.0)$$

corresponds to

$$(\nu x \bar{x})(x().\bar{x}\langle\rangle.(0 \mid 0))$$

which looks deadlocked to me.

Where to go from here?

We can submit to a nice, sunny conference upstate…

Where to go from here?

We can restrict the type system further, to restrict access to named channels if their co-names are still in scope...

(Dardha and Gay, 2017)

Where to go from here?

This gets rid of stuff like

$$(\nu x\bar{x})((\nu y\bar{y})(x().\bar{x}\langle\rangle.y().\bar{y}\langle\rangle.(0 \mid 0))).$$

(Dardha and Gay, 2017)

Where to go from here?

We can equate all these processes,

$$\pi.(P \mid Q) \equiv (\pi.P \mid Q) \quad \text{if} \quad \mathrm{fv}(\pi) \cap \mathrm{fv}(Q) = \varnothing$$

which allows all deadlocked processes to proceed.

(Bellin and Scott, 1994)

Basically, this is like saying

$$(\nu x\bar{x})(x().\bar{x}\langle\rangle.(0 \mid 0))$$

is equivalent to

$$(\nu x\bar{x})(x().0 \mid \bar{x}\langle\rangle.0),$$

so it can reduce.

(Bellin and Scott, 1994)

HCCP – Reduction Rules

$$
\begin{array}{lcl}
(\nu x\bar{x})(w \leftrightarrow x \mid P) & \Longrightarrow & P\{w/\bar{x}\} \\
(\nu x\bar{x})(x(z).R \mid \bar{x}\langle y\rangle.P) & \Longrightarrow & (\nu x\bar{x})(P \mid R) \\
(\nu x\bar{x})(x().R \mid \bar{x}\langle\rangle.P) & \Longrightarrow & (\nu x\bar{x})(P \mid R)
\end{array}
$$

$$
\frac{P \Longrightarrow P'}{(\nu x\bar{x})P \Longrightarrow (\nu x\bar{x})P'} \qquad \frac{P \Longrightarrow P'}{(P \mid Q) \Longrightarrow (P' \mid Q)}
$$

$$
\frac{P \equiv Q \qquad Q \Longrightarrow Q' \qquad Q' \equiv P'}{P \Longrightarrow P'}
$$

Where $\equiv$ is reflexive, transitive, congruent, and has:

$$
\begin{array}{lcll}
x \leftrightarrow y & \equiv & y \leftrightarrow x & \\
(P \mid Q) & \equiv & (Q \mid P) & \\
(P \mid (Q \mid R)) & \equiv & ((P \mid Q) \mid R) & \\
(\nu x \bar{x})(P \mid Q) & \equiv & ((\nu x \bar{x})P \mid Q) & \text{if } x, \bar{x} \notin \mathrm{fv}(Q) \\
\pi.(P \mid Q) & \equiv & (\pi.P \mid Q) & \text{if } \mathrm{fv}(\pi) \cap \mathrm{fv}(Q) = \varnothing
\end{array}
$$

I have found the calculus I was looking for, but maybe not the calculus I wanted…

Type

Ctxt

I'm gonna talk about hypersequent CP, about constrained cyclic Me

$$\frac{}{x \leftrightarrow y \;\vdash\; x : A, y : A^{\perp}}\ \textsc{Ax} \qquad \frac{}{0 \;\vdash\; \varnothing}\ \textsc{H-Halt}$$

$$\frac{P \;\vdash\; \mathcal{G} \mid \Gamma, x : A \mid \Delta, x : A^{\perp}}{(\nu x)P \;\vdash\; \mathcal{G} \mid \Gamma, \Delta}\ \textsc{H-Cut} \qquad \frac{P \;\vdash\; \mathcal{G} \qquad Q \;\vdash\; \mathcal{H}}{P \mid Q \;\vdash\; \mathcal{G} \mid \mathcal{H}}\ \textsc{H-Mix}$$

$$
\begin{array}{lll}
\text{Type} & A, B & := A \otimes B \mid A \parr B \mid \mathbf{1} \mid \bot \mid \ldots \\
\text{Ctxt} & \Gamma, \Delta & := x_1 : A_1, \ldots, x_n : A_n \\
\text{Meta} & X, Y & := \Gamma_1 \mid \ldots \mid \Gamma_n
\end{array}
$$

$$
\frac{P \vdash \mathcal{G} \mid \Gamma, y : A \mid \Delta, x : B}{x[y].P \vdash \mathcal{G} \mid \Gamma, \Delta, x : A \otimes B} \otimes
\qquad
\frac{P \vdash \mathcal{G} \mid \Gamma, y : A, x : B}{x(y).P \vdash \mathcal{G} \mid \Gamma, x : A \parr B} (\parr)
$$

$$
\frac{P \vdash \mathcal{G}}{x[].P \vdash \mathcal{G} \mid x : \mathbf{1}} \mathbf{1}
\qquad
\frac{P \vdash \mathcal{G} \mid \Gamma}{x().P \vdash \mathcal{G} \mid \Gamma, x : \bot} (\bot)
$$