A bunch of things to do with $\mathbf{NL}_\lambda$

Wen Kokke

December 7^{th}, 2016

What is $\mathbf{NL}_\lambda$?

Formula	A, B	$:= \alpha \mid A\backslash B \mid B/A$
Structure$^+$	Γ	$:= \cdot A \cdot \mid A \bullet B$
Structure$^-$	Δ	$:= \cdot A \cdot \mid A\backslash B \mid B/A$

$$\frac{}{\cdot\alpha\cdot \vdash \cdot\alpha\cdot}\ \text{Ax}$$

$$\frac{\Gamma \vdash \cdot A \cdot \qquad \cdot B \cdot \vdash \Delta}{\cdot A\backslash B \cdot \vdash \Gamma\backslash\Delta}\ \text{L}\backslash \qquad \frac{\Gamma \vdash \cdot A \cdot \backslash \cdot B \cdot}{\Gamma \vdash \cdot A\backslash B \cdot}\ \text{R}\backslash$$

$$\frac{\Gamma \bullet \Gamma' \vdash \Delta}{\Gamma \vdash \Gamma'\backslash\Delta}\ \text{Res}\bullet\backslash$$

What is $\mathbf{NL}_\lambda$?

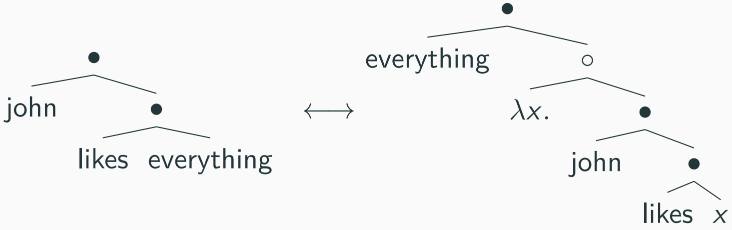
$$
\begin{array}{llll}
\text{Formula} & A, B & \coloneqq \ldots \mid A \mathbin{\backslash\!\!\backslash} B \mid B \mathbin{/\!\!/} A \\
\text{Structure}^{+} & \Gamma & \coloneqq \ldots \mid A \circ B \\
\text{Structure}^{-} & \Delta & \coloneqq \ldots \mid A \mathbin{\backslash\!\!\backslash} B \mid B \mathbin{/\!\!/} A \\
\text{Context} & \Sigma & \coloneqq \Box \mid \Sigma \bullet \Gamma \mid \Gamma \bullet \Sigma
\end{array}
$$

$$
\frac{\Sigma[\Gamma] \vdash \Delta}{\Gamma \circ \lambda x.\Sigma[x] \vdash \Delta}\,(\lambda)
$$

$$
\frac{\Gamma \vdash \cdot A \cdot \qquad \cdot B \cdot \vdash \Delta}{\cdot A \mathbin{\backslash\!\!\backslash} B \cdot \vdash \Gamma \mathbin{\backslash\!\!\backslash} \Delta}\,\mathrm{L}\backslash\!\!\backslash \qquad \frac{\Gamma \vdash \cdot A \cdot \mathbin{\backslash\!\!\backslash} \cdot B \cdot}{\Gamma \vdash \cdot A \mathbin{\backslash\!\!\backslash} B \cdot}\,\mathrm{R}\backslash\!\!\backslash
$$

$$
\frac{\Gamma \circ \Gamma' \vdash \Delta}{\Gamma \vdash \Gamma' \mathbin{\backslash\!\!\backslash} \Delta}\,\mathrm{Res}\circ\backslash\!\!\backslash
$$

So why this λ rule?

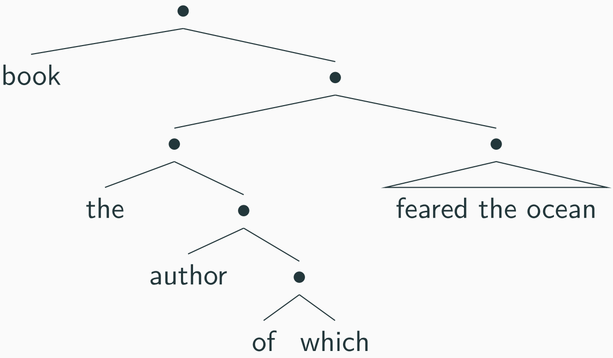


Why should we like $\mathbf{NL}_\lambda$?

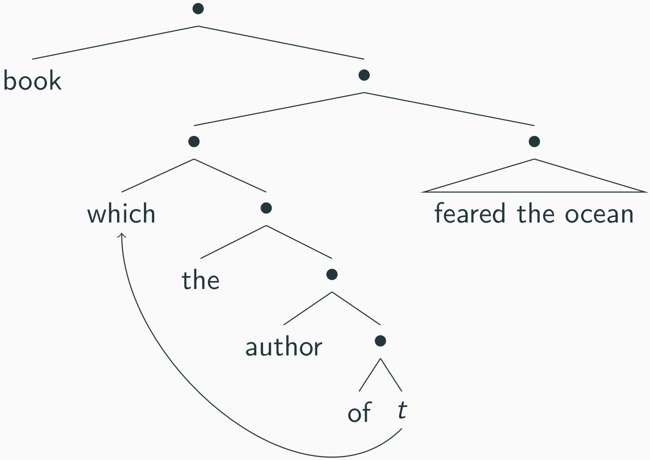
Example 1
"I read a book [the author of which] feared the ocean"

$$
\begin{aligned}
&\exists x.\mathbf{book}(x)\\
&\quad \wedge \mathbf{fear}(\iota(\lambda y.\mathbf{of}(y, \mathbf{author}, x)), \iota(\mathbf{ocean}))\\
&\quad \wedge \mathbf{read}(\mathbf{pepijn}, x)
\end{aligned}
$$

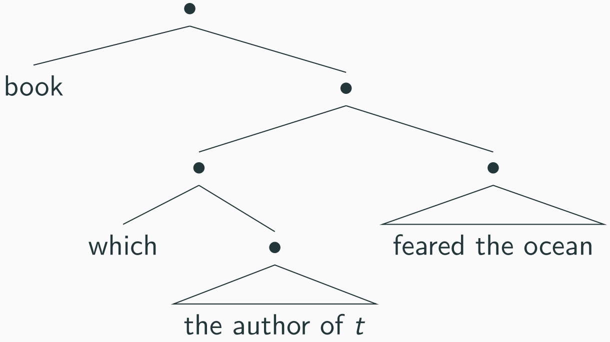
Why should we like $\mathbf{NL}_\lambda$?



Why should we like $\mathbf{NL}_\lambda$?



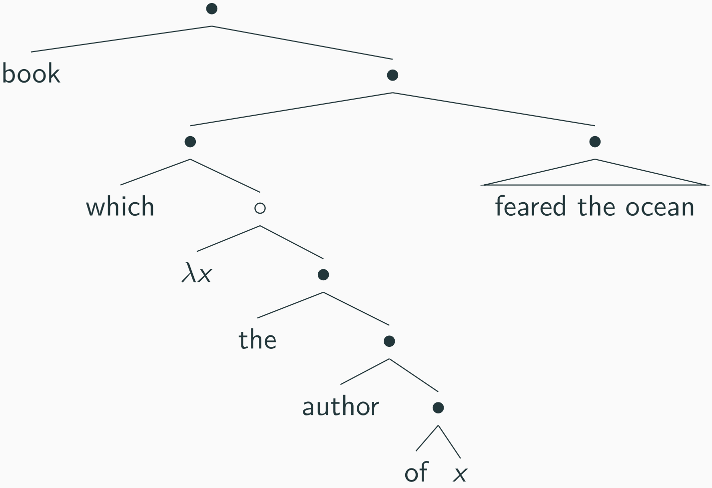
Why should we like NL$_\lambda$?



$$\text{which} \;:\; [\![np \,/\!\!/\, (np \,\backslash\!\!\backslash\, ((n\backslash n)/(np\backslash s)))]\!]$$

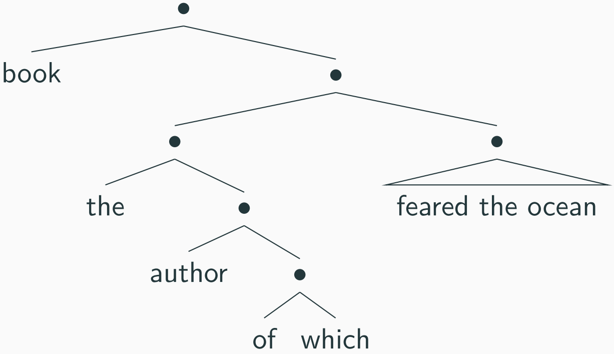
$$\text{which} = \lambda \text{tao}_{ee}.\lambda \text{fto}_{et}.\lambda \text{bk}_{et}.\lambda x_e.\text{bk}(x) \wedge \text{fto}(\text{tao}(x))$$

Why should we like NL_λ?

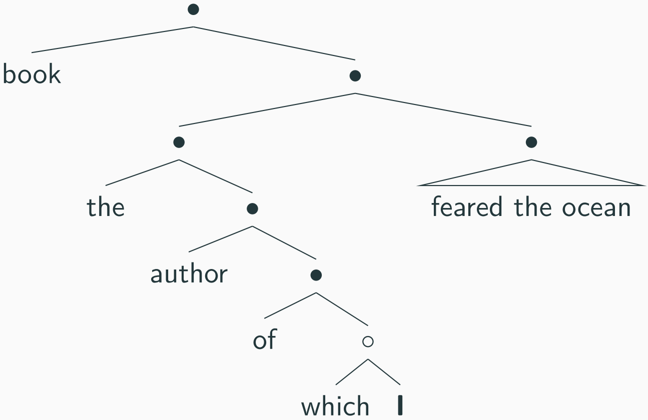


Take-home message
NL_λ gives us operational semantics for quantifier raising à la delimited continuations, without changing any other part of our type system.

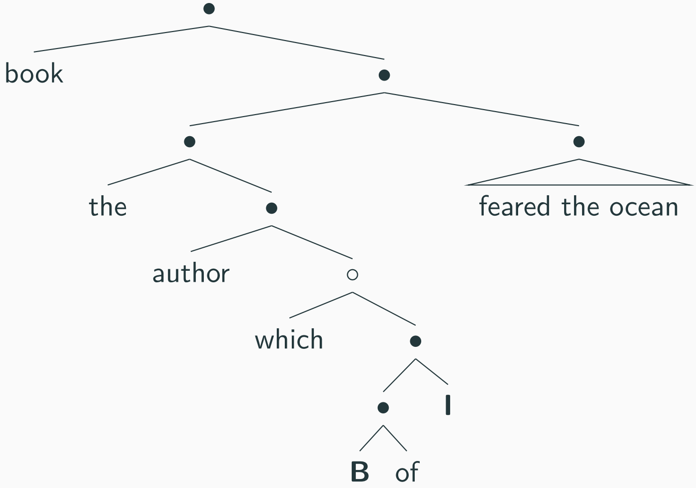
Why should we like $\mathbf{NL}_{CL}$?



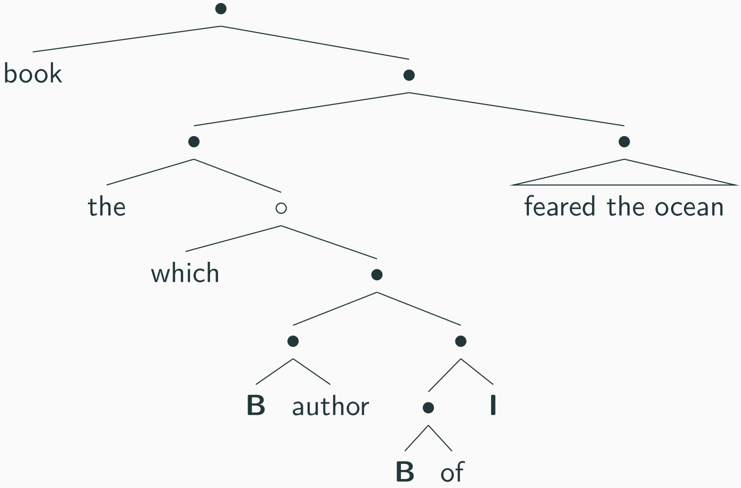
Why should we like $\mathbf{NL}_{CL}$?



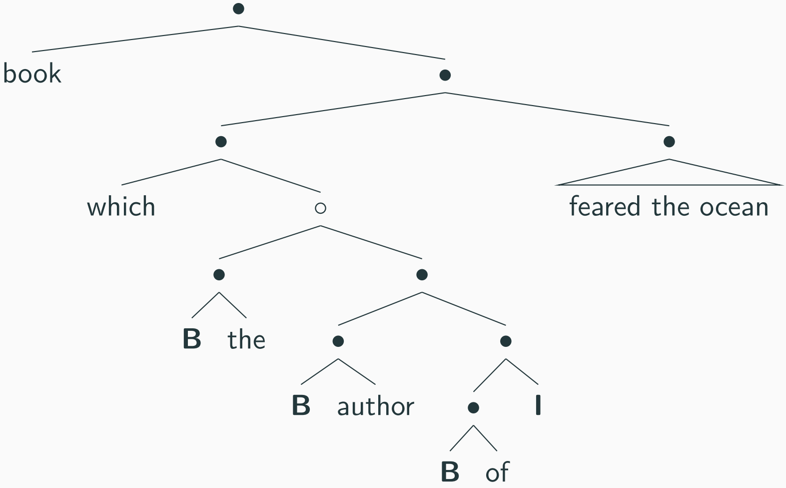
Why should we like $\mathbf{NL}_{CL}$?



Why should we like $\mathbf{NL}_{CL}$?



Why should we like NL_{CL}?



Why should we like $\mathbf{NL}_{CL}$?

$$\text{Structure}^{+}\ \Gamma \quad := \ldots \mid \mathbf{I} \mid \mathbf{B} \mid \mathbf{C}$$

$$\frac{\Gamma \vdash \Delta}{\Gamma \circ \mathbf{I} \vdash \Delta}\,\mathbf{I}$$

$$\frac{\Gamma_1 \bullet (\Gamma_2 \bullet \Gamma_3) \vdash \Delta}{\Gamma_2 \bullet ((\mathbf{B} \bullet \Gamma_1) \bullet \Gamma_3) \vdash \Delta}\,\mathbf{B} \qquad \frac{(\Gamma_1 \bullet \Gamma_2) \bullet \Gamma_3 \vdash \Delta}{\Gamma_1 \bullet ((\mathbf{C} \bullet \Gamma_2) \bullet \Gamma_3) \vdash \Delta}\,\mathbf{C}$$

How do we parse with $\mathbf{NL}_{CL}$?

What do we change?

We restrict quantifier raising s.t.

only quantifiers can be raised; and

only once.

We add focusing to eliminate spurious proofs.[1]

[1] Following work by Michael Moortgat, Raffaella Bernardi and Richard Moot (2012) and Arno Bastenhof (2011).

What does that look like?

$$\text{Context } \Sigma := \Box \mid \Sigma \bullet \Delta \mid \Gamma \bullet \Sigma$$

$$\begin{aligned} \Box[\Gamma'] &\mapsto \Gamma' & \overline{\Box} &\mapsto \mathbf{I} \\ (\Sigma \bullet \Gamma)[\Gamma'] &\mapsto (\Sigma[\Gamma'] \bullet \Gamma) & \overline{\Sigma \bullet \Gamma} &\mapsto ((\mathbf{C} \bullet \Sigma[\Gamma']) \bullet \Gamma) \\ (\Gamma \bullet \Sigma)[\Gamma'] &\mapsto (\Gamma \bullet \Sigma[\Gamma']) & \overline{\Gamma \bullet \Sigma} &\mapsto ((\mathbf{B} \bullet \Gamma) \bullet \Sigma[\Gamma']) \end{aligned}$$

$$\frac{\overline{\Sigma} \vdash \cdot B \cdot \qquad \cdot C \cdot \vdash \Delta}{\Sigma[\cdot C \mathbin{/\!\!/} B \cdot] \vdash \Delta}\,\mathsf{L}q \qquad \frac{\Sigma[\cdot A \cdot] \vdash \cdot B \cdot}{\overline{\Sigma} \vdash \cdot A \mathbin{\backslash\!\!\backslash} B \cdot}\,\mathsf{R}q$$

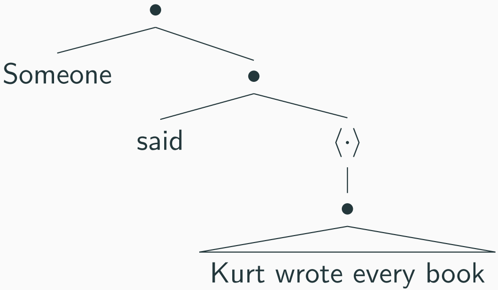
Take-home message
If you had any qualms about the decidability and efficiency of proof search with NL_λ, let them go, at least for the remainder of this talk.

Scope islands

Example 2
"Someone said $\langle$ Kurt wrote every book $\rangle$"

$$\exists x.\mathbf{person}(x) \wedge \mathbf{said}(x, \forall y.\mathbf{book}(y) \supset \mathbf{wrote}(\mathbf{kurt}, y))$$

Scope islands



said : $[\![(np\backslash s)/\Diamond s\ ]\!]$

said $= \ldots$

Not *That* Diamond and Box

$$
\begin{array}{lll}
\text{Formula} & A, B & \coloneqq \ldots \mid \Diamond A \mid \Box A \\
\text{Structure}^{+} & \Gamma & \coloneqq \ldots \mid \langle \Gamma \rangle \\
\text{Structure}^{-} & \Delta & \coloneqq \ldots \mid [\Delta]
\end{array}
$$

$$
\frac{\langle \cdot A \cdot \rangle \vdash \Delta}{\cdot \Diamond A \cdot \vdash \Delta}\,\mathsf{L}\Diamond
\qquad
\frac{\Gamma \vdash \cdot B \cdot}{\langle \Gamma \rangle \vdash \cdot \Diamond B \cdot}\,\mathsf{R}\Diamond
$$

$$
\frac{\cdot A \cdot \vdash \Delta}{\cdot \Box A \cdot \vdash [\Delta]}\,\mathsf{L}\Diamond
\qquad
\frac{\Gamma \vdash [\cdot B \cdot]}{\Gamma \vdash \cdot \Box B \cdot}\,\mathsf{R}\Box
$$

$$
\frac{\Gamma \vdash [\Delta]}{\langle \Gamma \rangle \vdash \Delta}\,\mathsf{Res}\Box\Diamond
$$

Take-home message
Things don't have to be difficult.

Indefinite scope

Example 3
"Everyone said ⟨ Kurt dedicated a book to Mary ⟩"

$$\forall x.\mathbf{person}(x) \supset \mathbf{said}(x, \exists y.\mathbf{book}(y) \wedge \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

$$\forall x.\mathbf{person}(x) \supset \exists y.\mathbf{book}(y) \wedge \mathbf{said}(x, \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

$$\exists y.\mathbf{book}(y) \wedge \forall x.\mathbf{person}(x) \supset \mathbf{said}(x, \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

“Indefinites acquire their existential scope in a manner that does not involve movement and is essentially syntactically unconstrained.”

— Anna Szabolcsi, The Syntax of Scope

Indefinite scope

Example 3
"Everyone said ⟨ Kurt dedicated a book to Mary ⟩"

$$\forall x.\mathbf{person}(x) \supset \mathbf{said}(x, \exists y.\mathbf{book}(y) \wedge \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

$$\forall x.\mathbf{person}(x) \supset \exists y.\mathbf{book}(y) \wedge \mathbf{said}(x, \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

$$\exists y.\mathbf{book}(y) \wedge \forall x.\mathbf{person}(x) \supset \mathbf{said}(x, \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

(ramble about continuations)

Continuation Semantics

$$\text{some} : \llbracket\ np/n\ \rrbracket$$
$$\text{some}\ (f, k)\ = \exists_e x.f(x) \wedge k(x) \tag{1}$$

$$\frac{\overline{\Sigma} \vdash \boxed{np \mathbin{\backslash\!\!\backslash} s} \qquad \boxed{s} \vdash \cdot s \cdot}{\Sigma[\cdot s \mathbin{/\!\!/} (np \mathbin{\backslash\!\!\backslash} s)\cdot] \vdash \cdot s \cdot}\ \mathsf{L}q \tag{2}$$

$$\frac{\dfrac{\text{Kurt dedicated} \ldots \ \vdash \boxed{\quad s}}{\langle \text{Kurt dedicated} \ldots \rangle \vdash \boxed{\Diamond s}}\ \mathsf{R}\Diamond \qquad \boxed{np\backslash s} \vdash \text{everyone}\backslash \cdot s \cdot}{\boxed{(np\backslash s)/\Diamond s} \vdash (\text{everyone}\backslash \cdot s \cdot)/\langle \text{Kurt dedicated} \ldots \rangle} \tag{3}$$

Indefinite scope

Example 3
"Everyone said $\langle$ Kurt dedicated a book to Mary $\rangle$"

$$\forall x.\mathbf{person}(x) \supset \mathbf{said}(x, \exists y.\mathbf{book}(y) \wedge \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

$$\forall x.\mathbf{person}(x) \supset \exists y.\mathbf{book}(y) \wedge \mathbf{said}(x, \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

$$\exists y.\mathbf{book}(y) \wedge \forall x.\mathbf{person}(x) \supset \mathbf{said}(x, \mathbf{dedicate}(\mathbf{kurt}, \mathbf{mary}, y))$$

What makes up a bunch?

- Display NL_{λ}
- (Parasitic scope, delimited continuations)
- Focusing and efficient proof search
- Scope islands
- Indefinite scope

A little bit of Haskell

$$no, every :: Word\ (\mathbf{Q}^W (((S \mathbin{/\!\!/} NP) \mathbin{\backslash\!\!\backslash} S)/N)$$
$$no \quad = lex\_\ (\lambda f\ g \rightarrow \neg\ (\exists_{\mathbf{e}}\ (\lambda x \rightarrow f\ x \wedge g\ x)))$$
$$every = lex\_\ (\lambda f\ g \rightarrow \forall_{\mathbf{e}}\ (\lambda x \rightarrow f\ x \supset g\ x))$$

$s22 = [nlq \mid mary\ reads\ a\ book\ (the\ author\ of\ which)\ john\ likes\ \mid]$

```
∃x0.(book x0 ∧ like john (the (λx1.(of x0 (λx2.(author x2))
    x1)))) ∧ read mary x0
```

$s23 = [nlq \mid mary\ sees\ foxes\ \mid]$

```
∃x0.(∃x1.(∃x2.x0 x1 ∧ x0 x2 ∧ x1 ≢ x2)) ∧ (∀x3.x0 x3 ⊃ (fox
    x3 ∧ see mary x3))
```

A little bit of Agda

```
qR : ∀ x → NLQ x [ · a · ] ⊢ · b · → NLQ trace(x) ⊢ · b // a ·
qR x f = impLR (resPL (↓ x f))
  where
  ↓ : x → NLQ x [ y ] ⊢ z → NLQ trace(x) ∘ y ⊢ z
  ↓ ( HOLE        ) f = unitLI f
  ↓ ( PROD1  x y  ) f = dnC (resLP (↓ x (resPL f)))
  ↓ ( PROD2  x y  ) f = dnB (resRP (↓ y (resPR f)))
```

What makes up a bunch?

- Display NL_λ
- (Parasitic scope, delimited continuations)
- Focusing and efficient proof search
- Scope islands
- Indefinite scope

Bonus Slides

What does focusing look like?

$$\begin{aligned} \mathsf{Pol}(np) &= + & \mathsf{Pol}(A\backslash B) &= - \\ \mathsf{Pol}(n) &= + & \mathsf{Pol}(B/A) &= - \\ \mathsf{Pol}(s) &= - \end{aligned}$$

$$\mathsf{Pos}(A) \iff \mathsf{Pol}(A) = + \qquad \mathsf{Neg}(A) \iff \mathsf{Pol}(A) = -$$

What does focusing look like?

$$\text{if } \mathsf{Pos}(\alpha) \left\{ \quad \frac{}{\cdot\alpha\cdot \vdash \boxed{\alpha}}\,\mathsf{Ax}^R \quad \middle| \quad \frac{}{\boxed{\alpha} \vdash \cdot\alpha\cdot}\,\mathsf{Ax}^L \quad \right\} \text{if } \mathsf{Neg}(\alpha)$$

$$\text{if } \mathsf{Pos}(A) \left\{ \begin{array}{c|c} \dfrac{\Gamma \vdash \boxed{A}}{\Gamma \vdash \cdot A\cdot}\,\mathsf{Foc}^R & \dfrac{\boxed{A} \vdash \Delta}{\cdot A\cdot \vdash \Delta}\,\mathsf{Foc}^L \\ \\ \dfrac{\cdot A\cdot \vdash \Delta}{\boxed{A} \vdash \Delta}\,\mathsf{Unf}^L & \dfrac{\Gamma \vdash \cdot A\cdot}{\Gamma \vdash \boxed{A}}\,\mathsf{Unf}^R \end{array} \right\} \text{if } \mathsf{Neg}(A)$$

What does focusing look like?

$$\frac{\Gamma \vdash \boxed{A} \qquad \boxed{B} \vdash \Delta}{\boxed{A \backslash B} \vdash \Gamma \backslash \Delta}\,\mathsf{L}\backslash \qquad \frac{\Gamma \vdash \boxed{A} \qquad \boxed{B} \vdash \Delta}{\boxed{B / A} \vdash \Delta / \Gamma}\,\mathsf{L}/$$

Continuation Semantics

$$s^* \mapsto \mathbf{t}, \qquad n^* \mapsto \mathbf{et}, \qquad np^* \mapsto \mathbf{e}, \qquad \ldots$$

$$\begin{aligned}
[\![\alpha]\!]^+ &\mapsto \begin{cases} \alpha^* & \text{if } \mathrm{Pos}(\alpha) \\ ((\alpha^*)^R)^R & \text{if } \mathrm{Neg}(\alpha) \end{cases} \\
[\![A \setminus B]\!]^+ &\mapsto ([\![A]\!]^+ \times [\![B]\!]^-)^R \\
[\![B \,/\, A]\!]^+ &\mapsto ([\![B]\!]^- \times [\![A]\!]^+)^R \\
[\![\Diamond A]\!]^+ &\mapsto [\![A]\!]^+ + \\
[\![\Box A]\!]^+ &\mapsto ([\![A]\!]^+ +)^R
\end{aligned}$$

(where $A^R := A \to \mathbf{t}$)

Continuation Semantics

$$s^* \mapsto \mathbf{t}, \qquad n^* \mapsto \mathbf{et}, \qquad np^* \mapsto \mathbf{e}, \qquad \ldots$$

$$\begin{aligned}
[\![\alpha]\!]^- &\mapsto (\alpha^*)^R \\
[\![A \setminus B]\!]^- &\mapsto [\![A]\!]^+ \times [\![B]\!]^- \\
[\![B \,/\, A]\!]^- &\mapsto [\![B]\!]^- \times [\![A]\!]^+ \\
[\![\Diamond A]\!]^- &\mapsto ([\![A]\!]^+ +)^R \\
[\![\Box A]\!]^- &\mapsto [\![A]\!]^+ +
\end{aligned}$$

(where $A^R := A \to \mathbf{t}$)

Continuation Semantics

$$s^* \mapsto \mathbf{t}, \qquad n^* \mapsto \mathbf{et}, \qquad np^* \mapsto \mathbf{e}, \qquad \ldots$$

$$\begin{aligned}
[\![\Gamma \vdash \Delta]\!] &\mapsto [\![\Gamma]\!] \vdash [\![\Delta]\!] \\
[\![\boxed{A} \vdash \Delta]\!] &\mapsto [\![\Delta]\!] \vdash [\![A]\!]^- \\
[\![\Gamma \vdash \boxed{A}]\!] &\mapsto [\![\Gamma]\!] \vdash [\![A]\!]^+
\end{aligned}$$

(where $A^R := A \rightarrow \mathbf{t}$)

References